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Erratum

Volume 11, Number 2, June (1974), in the article entitled, "The L_1 Norm of the Approximation Error for Splines with Equidistant Knots," by Wassily Hoeffding, pp. 176–193:

In the proof of Theorem 1, the demonstration of inequalities (3.14) is in error because they do not follow from (3.18), as claimed.

The portion of the paper from p. 184, line 6 ("From (2.9) ...") to p. 186, line 13 ("... inequality (1.8)"), should be deleted and replaced by the following.

It is sufficient to prove that

$$I_{m,k} \leqslant I_{m,k+1}, \qquad J_{m,k} \leqslant J_{m,k+1}, \qquad k = 1,..., m-2, \tag{1}$$

where, with $\xi_k = \xi(m, k) = \frac{1}{2}(k+1)k/(m-1)$,

$$I_{m,k} = L_k(\xi_{k-1}), \qquad J_{m,k} = L_k(\xi_k),$$

$$L_k(\xi) = \int | x - \xi | \, dH_{m-1,k}(x).$$

We can write (compare (2.9) and the last paragraph of Section 2)

$$L_k(\xi) = \iint | su - \xi | \, dG_{m-1,k+1}(u) \, h_k(s) \, ds,$$

$$L_{k+1}(\xi) = \iint | su + u - \xi | \, dG_{m-1,k+1}(u) \, h_k(s) \, ds.$$

Since $h_k(s) = h_k(k-s)$, these expressions are also true with s replaced by $k-s$. This implies the representation

$$L_{k+1}(\eta) - L_k(\xi) = \iint \phi\left(s - \frac{k}{2}, u, \xi, \eta\right) dG_{m-1,k+1}(u) \, h_k(s) \, ds,$$

where

$$2\phi(t, u, \xi, \eta) = \left| tu + \frac{k+2}{2} u - \eta \right| - \left| tu + \frac{k}{2} u - \xi \right| + \left| -tu + \frac{k+2}{2} u - \eta \right| - \left| -tu + \frac{k}{2} u - \xi \right|.$$

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We have $\phi(t, u, \xi_k, \xi_{k+1}) \geqslant 0$ for all t, u. This implies the second set of inequalities (1).

The remaining inequalities have to be proved by a different argument since $\phi(t, u, \xi_{k-1}, \xi_k) < 0$ for some t, u. Let

$$\psi(t) = \int \phi(t, u, \xi_{k-1}, \xi_k)\, dG_{m-1,k+1}(u).$$

Note that $\psi(t) = \psi(-t)$. It is sufficient to show that $\psi(t) \geqslant 0$ for $0 < t < k/2$.

We can express $\psi(t)$ in the form

$$\psi(t) = \xi_k K(\beta(t)) + \xi_k K(\beta(-t)) - \xi_{k-1} K(\alpha(t)) - \xi_{k-1} K(\alpha(-t)) + \int u\, dG(u) - \xi_k + \xi_{k-1},$$

where

$$K(x) = G(x) - x^{-1} \int_0^x u\, dG(u), \qquad G(x) = G_{m-1,k+1}(x),$$

$$\alpha(t) = \xi_{k-1} \left(\frac{k}{2} - t\right)^{-1}, \qquad \beta(t) = \xi_k \left(\frac{k+2}{2} - t\right)^{-1}.$$

The derivative of $\psi(t)$ is

$$\psi'(t) = -\int_0^{\beta(t)} u\, dG(u) + \int_0^{\beta(-t)} u\, dG(u) + \int_0^{\alpha(t)} u\, dG(u) - \int_0^{\alpha(-t)} u\, dG(u).$$

We have $\alpha(t) < \beta(t)$ for all $t \in (0, k/2)$, and $\alpha(-t) \leqslant \beta(-t)$ if and only if $t \leqslant \frac{1}{2}$. The function $K(x)$ is increasing for $x > 0$. Hence if $0 < t \leqslant \frac{1}{2}$ then $\psi(t) > 0$. If $\frac{1}{2} < t \leqslant k/2$ then $\psi'(t) \leqslant 0$. Thus $\psi(t) \geqslant 0$ for $t \in (0, k/2)$ if $\psi(k/2) = \lim_{t \uparrow k/2} \psi(t) \geqslant 0$, and the latter is readily proved. This completes the proof of (3.14) and, thus, of inequality (1.8).

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